Learning, evaluating and analyzing a recommendation rule for early blood transfer in the ICU

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Early blood transfer in the ICU

- Severely injured patients experiencing haemorrhagic shock often require massive transfusion
- Early transfusion of blood products (plasma, red blood cells and platelets) is common and associated with improved outcomes in the hospital
- However, determining a right amount of blood products is still a matter of scientific debate
- Our objective is to

build and *analyze* a *recommendation rule*

for *early transfusion of blood products*

to *assist* the medical team in the ICU

The Traumabase group

- Created in 2012, the French Traumabase group is a collaboration focusing on major trauma
- The group's objectives are to improve major trauma care, to inform public health decisions and to facilitate research
- Today,
  - 23 French trauma centers contribute to the Traumabase group
  - the Traumabase data set gathers information on more than 40,000 trauma cases from admission until discharge from the ICU all over France

The Traumabase data set

For each patient in the Traumabase data set, the information is naturally regrouped into four categories:

- epidemiological data:
  - age, gender, BMI, medical history, type of trauma (penetrating?), ...
- pre-hospital data:
  - Glasgow score, pupillary abnormality, blood pressures, maximum heart rate, minimum peripheral capillary oxygen saturation (SpO2), ...
- admission data:
  - same as above & differences pre-hospital vs. admission & temperature, indicator of hemorrhagic shock & early transfusion of blood products, ...
- hospital data:
  - durations of stay in the ICU, in the hospital; survival in the hospital

Early transfusion of blood products

- Blood products:
  - plasma (PFC), red blood cells (CGR), platelets
  - rare, and conserved frozen; need up to 45 minutes to unfreeze
- Despite official recommendations, *e.g.*
  - Société Française d'Anesthésie Réanimation (2015): ratio PFC/GCR between 1 and 2
  - European 5th guideline on management of major bleeding and coagulopathy following trauma (2019): ratio PFC/GCR 2

  one observes a significant variability in early transfusion of blood products practice
- What could be a best practice is still a matter of scientific debate

A recommendation rule: what is it?

- Statistical model:
  each patient contributes a data-structure $O := (W, T, Y) \sim P$ with
  - $W \in \mathcal{W}$: covariates (epidemiological data, pre-hospital data, admission data obtained before the early transfusion of blood products)
  - $T := (A, B, C) \in \mathcal{T}$: quantification of early transfusion of plasma (A), red blood cells (B) and platelets (C)
  - $Y \in \{0, 1\}$: indicator of survival in the hospital

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  - $Y \in \{0, 1\}$: indicator of survival in the hospital
- Causal model:
  each patient contributes a counterfactual data-structure $\mathbb{O} := (W, (Y[t])_{t \in \mathcal{T}}, T) \sim \mathbb{P}$ with
  - $Y[t] \in \{0, 1\}$: the *potential*/*counterfactual* indicator of survival in the hospital in a world where $T = t$ would be *imposed*
  - $Y := Y[T]$: the *actual* indicator of survival in the hospital, in the real world ("consistency assumption")
  - the set of optimal actions for *that* patient: $\arg\max_{t \in \mathcal{T}} Y[t]$
  - the set of optimal *recommendation rules* at the population level:

$$\arg\max_{r:\mathcal{W} \to \mathcal{T}} E_{\mathbb{P}}\{Y[r(W)]\}$$

A more realistic notion of recommendation rule

- Problem:
  - viewing the data set as a collection of $O_1, \ldots, O_i := (W_i, T_i, Y_i), \ldots, O_N \overset{\text{ind}}{\sim} P$, the empirical law of T is highly concentrated around $\{(a, b, c) = (q, q, q) : q \in \mathbb{N}\} \cap \mathcal{T}$ (*i.e.*, same number of bags for all blood products)
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  - hence would be unrealistic to look for a rule $r : \mathcal{W} \to \mathcal{T}$
- Proposed solution:
  - define $\tilde{A} := \mathbf{1}\{A = 1\} + 2\mathbf{1}\{A \geq 2\}$
  - instead of $\arg\max_{r:\mathcal{W}\to\mathcal{T}} E_{\mathbb{P}}\{Y[r(W)]\}$, target

$$\boxed{r^\star(\mathbb{P}) := \underset{r:\mathcal{W}\to\{0,1,2\}}{\arg\max} E_{\mathbb{P}}\{Y[\tilde{T}_r]\},} \qquad \text{where} \qquad \tilde{T}_r | W \sim \mathcal{L}_P(T|\tilde{A} = r(W), W)$$

Identifiability and learning strategy

- Is $r^\star(\mathbb{P})$ (a feature of $\mathbb{P}$) identifiable under P (*i.e.*, a feature of P)?

Identifiability and learning strategy

- Is $r^\star(\mathbb{P})$ (a feature of $\mathbb{P}$) identifiable under P (*i.e.*, a feature of P)?
  - if there exists $\delta > 0$ such that $\delta \leq P(\tilde{A} = \tilde{a}|W)$ for all $\tilde{a} \in \{0, 1, 2\}$ P-as. (testable)
  - if $T \perp Y[t]|W$ for all $t \in \mathcal{T}$ ("randomization assumption", untestable)
  - then, for every $r : \mathcal{W} \to \{0, 1, 2\}$,

$$E_{\mathbb{P}}\{Y[\tilde{T}_r]\} = E_P\{\bar{Q}_P(r(W), W)\}, \qquad \text{where} \qquad \bar{Q}_P(\tilde{A}, Y) := E_P(Y|\tilde{A}, W)$$

  hence

$$r^\star(\mathbb{P})(W) = \boxed{\arg\max_{\tilde{a} \in \{0,1,2\}} \bar{Q}_P(\tilde{a}, W) =: \rho(P)(W)}$$

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$$r^\star(\mathbb{P})(W) = \boxed{\arg\max_{\tilde{a}\in\{0,1,2\}} \bar{Q}_P(\tilde{a}, W) =: \rho(P)(W)}$$

- Consequence: an estimator $\bar{Q}_n$ of $\bar{Q}_P$ yields an estimator r_n of $\rho(P)$,

$$r_n : w \mapsto \arg\max_{\tilde{a}\in\{0,1,2\}} \bar{Q}_n(\tilde{a}, w)$$

Learning a recommendation rule (1/3)

The missSuperLearner R-package

- Building an estimator $\bar{Q}_n$ of $\bar{Q}_P$ is a regression task
- There exist many (many (many)) algorithms to learn $\bar{Q}_P$
- Instead of choosing one algorithm, we learn which one performs best for the task at hand
- We rely on super learning (van der Laan et al, 2007, $\{\ldots\}$):
  - an *aggregation procedure* based on cross-validation and the choice of a loss function tailored to the task at hand
  - where the candidate algorithms write as $\mathcal{Q} = \text{Algo}_{\text{regression}} \circ \text{Algo}_{\text{screening}}$

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- We adapt the `SuperLearner` R-package (Polley et al., 2021) and propose the `missSuperLearner` R-package
  - which deals with missing data
  - where the candidate algorithms write as $\mathcal{Q} = \text{Algo}_{\text{regression}} \circ \text{Algo}_{\text{screening}} \circ \text{Algo}_{\text{filling in}}$

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  - where the candidate algorithms write as $\mathcal{Q} = \text{Algo}_{\text{regression}} \circ \text{Algo}_{\text{screening}} \circ \text{Algo}_{\text{filling in}}$
- A question remains: which loss function?

Learning a recommendation rule (2/3)

Value of a recommendation rule

- Given a recommendation rule $r : \mathcal{W} \to \{0, 1, 2\}$, the value of r is

$$\boxed{\mathcal{V}_r(P) := E_P\{\bar{Q}_P(r(W), W)\}}$$

  - note: $\mathcal{V}_r(P) \leq \mathcal{V}_{\rho(P)}(P)$ by definition of $\rho(P)$

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  - archetypal example: in a regression framework, the least-squares loss function

$$\bar{Q} \mapsto \ell_2(\bar{Q}) \quad \text{s.t.} \quad \ell_2(\bar{Q}) : (w, t, y) \mapsto (y - \bar{Q}(\tilde{a}, w))^2$$

    induces the least-squares risk $\mathcal{R}_P^{\ell_2} : \bar{Q} \mapsto E_P\{\ell_2(\bar{Q})(O)\}$

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  - by analogy, the loss function

$$r \mapsto \ell_{\bar{Q}}(r) \quad \text{s.t.} \quad \ell_{\bar{Q}}(r) : (w, t, y) \mapsto -\bar{Q}(r(w), w)$$

    induces the negative-value risk $\mathcal{R}_P^{\ell_{\bar{Q}}} : r \mapsto E_P\{\ell_{\bar{Q}}(r)(O)\}$
    - note: if $\bar{Q} = \bar{Q}_P$, then $\mathcal{R}_P^{\ell_{\bar{Q}}} = -\mathcal{V}_r(P)$
    - challenge: the loss function is indexed by the nuisance parameter $\bar{Q}$

Learning a recommendation rule (3/3)

A tailored cross-validated risk

- Let $B_n \in \{0, 1\}^n$ be a random vector drawn independently of $O_1, \ldots, O_n$, $n := 0.7 \times N$, and
  - P_n, the data set $\{O_1, \ldots, O_n\}$ (viewed as a prob. measure)
  - P^0_{n,B_n}, the B_n-specific training data set $\{O_i : 1 \leq i \leq n \text{ s.t. } B_n(i) = 0\}$ (viewed as a prob. measure)
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- Inspired by the negative-value risk $\mathcal{R}_P^{\ell_{\bar{Q}}} : r \mapsto E_P\{\ell_{\bar{Q}}(r)(O)\}$, we choose the following cross-validated risk: for any algorithm Algo designed to learn $\bar{Q}_P$,

$$\mathcal{R}_{P,n}(\text{Algo}) := E_{B_n}\left[E_{P^1_{n,B_n}}\left\{\ell_{\bar{Q}^*_n}(r^0_n)(O)\right\}\right]$$

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training data

cross-validation

testing data

optimal fluctuation of $\bar{Q}^0_n$ targeting $\mathcal{V}_{r^0_n}(P)$

rule derived from $\bar{Q}^0_n := \text{Algo}(P^0_{n,B_n})$

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- The "discrete" super learner: $\text{Algo}_{\text{SL}} := \arg\min_{\text{Algo}} \mathcal{R}_{P,n}(\text{Algo})$
- Our recommendation rule: the rule r_n derived from $\bar{Q}_n = \text{Algo}_{\text{SL}}(P_n)$

Estimating the value of r_n

- From now on, we consider r_n as a fixed recommendation rule
- What is its value $\mathcal{V}_{r_n}(P)$? We estimate it by CV-TMLE (Zheng & van der Laan, 2011), using $O_{n+1}, \ldots, O_N$

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testing data

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output of super learner trained on P^0_{N,B_N}

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- Under mild assumptions, ν_N is asymptotically Gaussian with an asymptotic variance that can be conservatively estimated

Variable importance measures (1/2)

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$$\Psi_s(P) := \frac{E_P\left\{[r_n(W) - E_P(r_n(W)|W_{-s})]^2\right\}}{\mathrm{Var}_P(r_n(W))} \in [0,1]$$

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  - if $\Psi_s(P) = 0$, then W_s plays no role in $r_n(W)$
  - if $\Psi_s(P) \neq 0$, then we can build a consistent, asymptotically Gaussian estimator $\psi_{N,s}$ of $\Psi_s(P)$
  - if $\Psi_s(P) = 0$, then $\psi_{N,s}$ is no longer asymptotically Gaussian!
  - testing "$\Psi_s(P) = 0$" against "$\Psi_s(P) \neq 0$" is not easy...

Variable importance measures (2/2)

Testing

- Inspired by (Hudson et al., 2022) we note that, for every $W_s \subset W$,

$$\text{if } \Psi_s(P) = 0, \text{ then for each } h : \mathcal{W} \to \mathbb{R},$$

$$\Phi_{s,h}(P) := E_P \left\{ \left[r_n(W) - E_P(r_n(W)|W_{-s}) \right] h(W) \right\} = 0$$

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- So, given a RKHS and its approximation $\mathcal{H}$ generated by the K first eigenfunctions (with a constraint on the norms of its elements), we decide to test

$$H_{0,s} : \text{“} \sup_{h \in \mathcal{H}} \Phi_{s,h}(P) = 0\text{”} \quad \text{against} \quad \text{“} \sup_{h \in \mathcal{H}} \Phi_{s,h}(P) \neq 0\text{”}$$

and to reject “$\Psi_s(P) = 0$” for “$\Psi_s(P) \neq 0$” if we reject $H_{0,s}$ for $\neg H_{0,s}$

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- Using influence-curve-based estimators of any $\Phi_{s,h}(P)$ ($h \in \mathcal{H}$), we can build an estimator ω_N of $\sup_{h \in \mathcal{H}} \Phi_{s,h}(P)$ by solving an optimization programme

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$$\text{if } \Psi_s(P) = 0, \text{ then for each } h : \mathcal{W} \to \mathbb{R},$$
$$\Phi_{s,h}(P) := E_P \left\{ [r_n(W) - E_P(r_n(W) | W_{-s})] \, h(W) \right\} = 0$$

- So, given a RKHS and its approximation $\mathcal{H}$ generated by the K first eigenfunctions (with a constraint on the norms of its elements), we decide to test

$$H_{0,s} : \text{“} \sup_{h \in \mathcal{H}} \Phi_{s,h}(P) = 0\text{”} \quad \text{against} \quad \text{“} \sup_{h \in \mathcal{H}} \Phi_{s,h}(P) \neq 0\text{”}$$

  and to reject "$\Psi_s(P) = 0$" for "$\Psi_s(P) \neq 0$" if we reject $H_{0,s}$ for $\neg H_{0,s}$

- Using influence-curve-based estimators of any $\Phi_{s,h}(P)$ ($h \in \mathcal{H}$), we can build an estimator ω_N of $\sup_{h \in \mathcal{H}} \Phi_{s,h}(P)$ by solving an optimization programme
- A multiplier bootstrap procedure allows to approximate the law of ω_N under the null

Variable importance measures (2/2)

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- Using influence-curve-based estimators of any $\Phi_{s,h}(P)$ ($h \in \mathcal{H}$), we can build an estimator ω_N of $\sup_{h \in \mathcal{H}} \Phi_{s,h}(P)$ by solving an optimization programme
- A multiplier bootstrap procedure allows to approximate the law of ω_N under the null
- Thus we can test $H_{0,s}$ against $\neg H_{0,s}$, and provide p-values

Discussion

- Work still in progress:

  theory ✓ coding ✓ simulation study ✓ real data cleaning ✓ real data application ongoing

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- A few insights:
  - the empirical probability of survival equals $\sim$90%: difficult to perform as well
  - in ABC (more severe) patients, the empirical probability of survival equals $\sim$70%: perhaps less challenging to do as well or even better
  - we rely on $\sim$20 $\text{Algo}_{\text{regression}}$, $\sim$5 $\text{Algo}_{\text{screening}}$, 3 $\text{Algo}_{\text{filling in}}$
  - results of simulation study encouraging
  - choice of $\mathcal{H}$ is a delicate matter (chose $K = 20$)
  - in preliminary results,
    - the estimated values of the rules were quite close to one another
    - across the validation data, the recommendation rule's suggestions differed in law from the actual interventions

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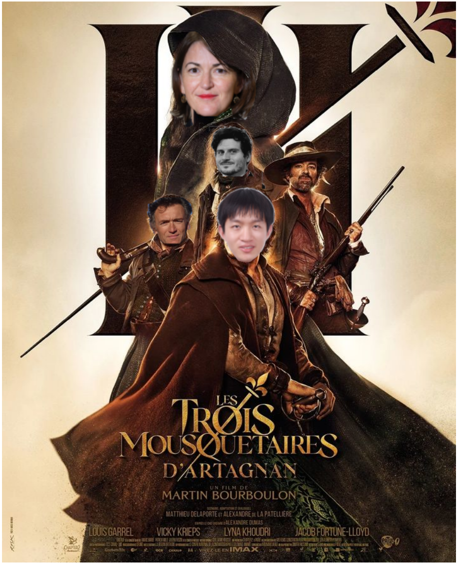
- Thank you very much for your attention. Any question?


LES
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UN FILM DE
MARTIN BOURBOULON
MATTHIEU DELAPORTE ET ALEXANDRE DE LA PATELLIÈRE
LOUIS GARREL
VICKY KRIEPS
LYNA KHOUDRI
JACOB FORTUNE-LLOYD
IMAX

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